Dispersion of interacting spinor cavity polaritons out of thermal equilibrium

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Using spinor Gross-Pitaevskii equations, we analyze the dispersion of elementary excitations of a polariton macro-occupied mode quasiresonantly excited in a microcavity. In a general case, the dispersion contains flat, purely dissipative parts characteristic for driven-dissipative systems. However, a Bogoliubov-like linear dispersion is found when the detuning between the laser energy and the bare energy of the pumped state is exactly compensated by the interparticle interaction. In this regime, the ensemble of polaritons demonstrates propagation with zero mechanical viscosity.

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I. INTRODUCTION

Cavity polaritons are elementary excitations of semiconductor microcavities in the strong coupling regime. Being linear combinations of quantum well (QW) excitons and cavity photons, they possess a number of peculiar properties which distinguish them from other quasiparticles in mesoscopic systems and make possible their applications in optoelectronics.[1–5]

The presence of the photonic component results in extremely small effective mass of cavity polaritons[6] (10^{-4}–10^{-5} of the electron mass), while the excitonic component makes possible efficient polariton-polariton interactions.[7] Under nonresonant pumping, this makes possible the thermalization of the polaritonic system[8] and can result in the Bose-Einstein condensation (BEC) of the exciton-polaritons[9,10] and superfluidity.[11–13] Under resonant pumping, polariton-polariton interactions lead to the possibility of the observation of a variety of intriguing nonlinear effects such as bistability of the pumped mode[14–18] and polariton parametric scattering.[17–24]

An important property of the cavity polaritons is their spin, inherited from the spin of the QW excitons. The ground state of the QW exciton has four possible spin projections on the structure growth axis: ±1, ±2. According to the selection rules, the dark states ±2 are not coupled with cavity mode. On the other hand, the states ±1 are coupled with photons and thus participate in the formation of the polariton doublet. They can be created by σ^+ and σ^- circularly polarized light, respectively. A linearly polarized light excites a linear combination of +1 and −1 exciton states, so that the total exciton spin projection on the structure axis is zero in this case, while the exciton polarization has a nonzero in-plane projection.

As the spin of polaritons is directly connected with the polarization of the emitted photons, the analysis of the latter is a powerful tool for the experimental investigation of spin dynamics of cavity polaritons.[25–31] The external magnetic fields together with effective internal magnetic fields of various origins lead to the mixing of different polarization components of polaritons. At $k \neq 0$, the circularly polarized components are normally mixed by strong TE-TM splitting of the photonic mode,[32,33] which can lead to numerous interesting phenomena such as the optical spin Hall effect[34–36] or the formation of polarization vortices.[37] On the other hand, anisotropy of the polariton-polariton interactions contributes to the mixing of the linearly polarized components.[38,39] Namely, the interaction of the polaritons in triplet configuration (parallel spin projections on the structure growth axis) is different from polaritons in singlet configuration (antiparallel spin projections on the structure growth axis). The interplay between spin and many-body interactions makes spin dynamics of the polaritons extremely rich and interesting and leads to some remarkable nonlinear polarization effects, such as self-induced Larmor precession (Faraday rotation)[27,40,41] and inversion of linear polarization during the scattering act.[42]

The goal of the present paper is to analyze the properties of a semiconductor microcavity under cw pumping, taking into account both spinor structure of the polaritons and anisotropic polariton-polariton interactions. In existing theoretical models, the external field intensity is usually assumed to be the only driving parameter of the system.[17,18] Such an assumption corresponds to the hypothetical situation of circularly polarized excitation without any further mixing of polarization. The description of the elliptical or linearly polarized excitation as well as accounting for TE-TM splitting cannot be carried out within the frameworks of the scalar model, making necessary the generalization of the model to take into account the spinor nature of cavity polaritons explicitly.

In our recent paper, we have extended the scalar semiclassical approach based on the Gross-Pitaevskii equation to account for two polarization states of resonantly driven cavity polaritons.[43] It was shown that the interplay between the nonlinearity caused by the polariton-polariton interactions and the polarization dependence of these interactions results in a multistability of the driven polariton mode, contrary to the usual optical bistability for the spinless nonlinear case.

We demonstrated that under coherent cw pumping at $k=0$ for a given polarization of the pump, the polariton polarization can, in general, take three different values.

In the present paper, we develop the approach proposed in Ref. 43 to calculate the dispersion of elementary excitations in a microcavity under resonant pumping. It is well known that for the system of scalar interacting bosons with infinite lifetime in thermodynamic equilibrium, the macroscopic occupation of the ground state leads to the renormalization of the spectrum of elementary excitations.[44,45] In the region near $k=0$, the dispersion consists of a linear Bogoliubov mode $E(k)=v_s k$, indicating the presence of superfluidity with a critical velocity v_s. The spinor nature of cavity polaritons and their finite lifetime make this picture more complicated.[12,46] Due to the anisotropy of polariton-polariton interactions, the condensate is expected to be linearly polarized. The dispersion of elementary excitations consists of two Bogoliubov-like modes with sound velocity depending on polarization.[12]

However, experimental measurements of the renormalization of the polariton dispersion in the regime of polariton BEC[47] did not reveal the appearance of the Bogoliubov mode. Instead, the renormalized dispersion was shown to flatten in the region near $k=0$. Two alternative explanations have been given to this observation so far. The first one is connected with the dynamical nature of the polariton BEC, where the processes of incoherent pumping and radiative decay compensate each other.[48–50] Alternatively, the appearance of the flat part was explained as resulting from the localization of cavity polaritons in disorder potential and phase transition toward the Bose glass phase.[51]

In the present paper, we analyze the dispersion of elementary excitations in a resonantly pumped microcavity. In this case, the macroscopic occupation of the ground state is achieved not due to the processes of thermalization but is built up directly by an external laser beam. The situation is thus very different from those considered in Refs. 47–49 and 51. The crucial point is that the energy and polarization of the macroscopically occupied mode are no more determined by the system itself in order to minimize the free energy but are rigidly fixed by the frequency and polarization of the coherent excitation. Consequently, the introduction of the chemical potential of the condensate is no more possible and one should expect, in general, that the dispersion of elementary excitations will differ from Bogoliubov-like modes. However, we found that for some excitation conditions, the spectrum of elementary excitations becomes linear and verifies the Landau criterion of superfluidity. These results, in agreement with the precedent work of Ciuti and Carusotto,[11] show that despite of the presence of pumping and decay, the gas of interacting polaritons in microcavities can form a superfluid. However, this superfluid is composed of radiatively decaying particles, thus differing from ordinary superfluids. Only dissipation of momentum by mechanical viscosity is suppressed, but the particles can still disappear due to the radiative decay.

This paper is organized as follows. In Sec. II, we introduce the spinor Gross-Pitaevskii equations for resonantly pumped polaritonic systems. In Sec. III, we obtain analytical expressions for the dispersions of elementary excitations for different polarizations of the pump and show the different possible dispersions. In Sec. IV, we analyze the stability conditions for the system under study. In Sec. V, we discuss the effects of dispersion renormalization on the emission spectra of a microcavity in pump-probe experiments.

II. MODEL

To get the dynamic equations for polaritonic field, we start with the Hamiltonian of the system which in the exciton-photon basis can be represented in the following form:

$$\begin{aligned}\hat{H} = \int d\mathbf{x} \sum_{j=\uparrow,\downarrow} [\hat{\psi}_X^{(j)+} E_X(\hat{\mathbf{k}}) \hat{\psi}_X^{(j)} + \hat{\psi}_{ph}^{(j)+} E_{ph}(\hat{\mathbf{k}}) \hat{\psi}_{ph}^{(j)}] \\ + V_R \sum_{j=\uparrow,\downarrow} [\hat{\psi}_X^{(j)+} \hat{\psi}_{ph}^{(j)} + \hat{\psi}_X^{(j)} \hat{\psi}_{ph}^{(j)+}] \\ + \frac{1}{2} \sum_{j=\uparrow,\downarrow} [W_1 \hat{\psi}_X^{(j)+} \hat{\psi}_X^{(j)+} \hat{\psi}_X^{(j)} \hat{\psi}_X^{(j)} + W_2 \hat{\psi}_X^{(j)+} \hat{\psi}_X^{(j)} \hat{\psi}_X^{(\bar{j})+} \hat{\psi}_X^{(\bar{j})}] \\ + \sum_{j=\uparrow,\downarrow} [P_j(\mathbf{k}) e^{i\omega_0 t} \hat{\psi}_{ph}^{(j)} + P_j^*(\mathbf{k}) e^{-i\omega_0 t} \hat{\psi}_{ph}^{(j)+}]. \qquad (1)\end{aligned}$$

Here, $\psi_{ph}^j(k,t)$ and $\psi_X^j(k,t)$ are the field operators of the cavity photons and QW excitons with indices describing two circular polarization states of the emitted light, $j=\uparrow,\downarrow;\bar{\uparrow}=\downarrow;\bar{\downarrow}=\uparrow$. $E_X(\hat{\mathbf{k}})$ and $E_{ph}(\hat{\mathbf{k}})$ are the bare energies of excitonic and photonic modes which can have nonzero imaginary parts due to their finite lifetimes. V_R is a half of the Rabi splitting and W_1 and W_2 are effective constants of exciton-exciton interaction in triplet and singlet configurations respectively. $P_j(\mathbf{k})$ is the amplitude of the pumping field with in-plane momentum $\mathbf{k}$ and frequency ω_0.

The first term in Eq. (1) describes the free excitons and cavity photons, the second term describes linear coupling between excitonic and photonic modes, the third term describes exciton-exciton interactions in s-wave approximation, and the fourth term corresponds to the coupling of the cavity mode with an external coherent pumping field.

Both exciton and photon are assumed to be ideal bosons, $[\hat{\psi}_X^{(j)+}(\mathbf{x});\hat{\psi}_X^{(j)}(\mathbf{x}')]=\delta(\mathbf{x}-\mathbf{x}')$, the effects of the saturation of the excitonic resonance thus being neglected. The latter can be taken into account by renormalization of V_R with excitonic density.[52,53]

Although the formalism we use can be easily generalized for an arbitrary microcavity excitation geometry, in the present paper, we assume that the original exciton and photon modes are twofold degenerate. This is the case for highly symmetrical microcavities excited by the pump beam normal to the surface. In the linear regime in such systems, polarization state of the internal field coincides with that of the pump, which also gives a good reference in the nonlinear regime. Note that in realistic microcavities, the cylindrical symmetry is often broken which results in splittings of the exciton and photon modes (see Refs. 4, 12, and 47). We neglect this effect here.

Using the Heisenberg equation of motion $i\partial_t \hat{\psi}_{X,ph}^{(j)} = [\hat{\psi}_{X,ph}^{(j)};\hat{H}]$ $(\hbar=1)$, one can obtain the system of four coupled nonlinear equations for the excitonic and photonic fields,

$$\left[i\frac{\partial}{\partial t}-E_{ph}(-i\nabla)\right]\Psi_{ph}^{(j)}(\mathbf{x})=V_R\Psi_X^{(j)}(\mathbf{x})+Pe^{-i\omega_0 t},\qquad(2)$$

$$\left[i\frac{\partial}{\partial t}-E_X(-i\nabla)\right]\Psi_X^{j}(\mathbf{x})=V_R\Psi_{ph}^{(j)}(\mathbf{x})+[W_1|\Psi_X^{(j)}(\mathbf{x})|^2+W_2|\Psi_X^{(\bar{j})}(\mathbf{x})|^2]\Psi_X^{(j)}(\mathbf{x}),\qquad(3)$$

where c functions of the bosonic fields were introduced as

$$\Psi_{ph,X}^{(j)}(\mathbf{x})=\langle\hat{\psi}_{ph,X}^{(j)}\rangle.\qquad(4)$$

Passing from Eq. (1) to Eqs. (2) and (3), we used the mean-field approximation

$$\langle\hat{\psi}_X^{(j)+}\hat{\psi}_X^{(j)}\hat{\psi}_X^{(j)}\rangle\approx|\Psi_X^{(j)}|^2\Psi_X^{(j)}.\qquad(5)$$

Equations (2) and (3) correspond to the nonlinear Schrödinger equation describing the optical media with $\chi^{(3)}$ nonlinearity. It should be noted that although it gives an accurate description of the nonlinear properties of quantum microcavities in the strong pumping regime and accounts for the stimulated polariton-polariton scattering, certain aspects of the polariton dynamics, e.g., scattering with acoustic phonons and spontaneous polariton-polariton scattering, are not accounted for by Eqs. (2) and (3). These scattering processes can be treated as dissipative perturbations, which can modify the system dynamics but not its dispersion which we are interested in.

To make possible the analysis of the response of the system, it is convenient to rewrite Eqs. (2) and (3) in the polariton basis. The procedure is as follows. Let us rewrite Eqs. (2) and (3) in $\mathbf{k}$ representation

$$\left[i\frac{\partial}{\partial t}-E_{ph}(\mathbf{k})\right]\Psi_{ph}^{(j)}(\mathbf{k})=V_R\Psi_X^{(j)}(\mathbf{k})+\delta(\mathbf{k})Pe^{-i\omega_0 t},\qquad(6)$$

$$\left[i\frac{\partial}{\partial t}-E_X(\mathbf{k})\right]\Psi_X^{j}(\mathbf{k})=V_R\Psi_{ph}^{(j)}(\mathbf{k})+W_1\int\Psi_X^{(j)*}(\mathbf{k}')\times\Psi_X^{(j)}(\mathbf{k}-\mathbf{q})\Psi_X^{(j)}(\mathbf{k}'+\mathbf{q})d\mathbf{k}'d\mathbf{q}+W_2\int\Psi_X^{(\bar{j})*}(\mathbf{k}')\Psi_X^{(\bar{j})}(\mathbf{k}-\mathbf{q})\times\Psi_X^{(j)}(\mathbf{k}'+\mathbf{q})d\mathbf{k}'d\mathbf{q},\qquad(7)$$

and then introduce upper and lower polariton amplitudes by means of the unitary transformation

$$\Psi_{LP}^{(j)}(\mathbf{k})=C_L(\mathbf{k})\Psi_{ph}^{(j)}(\mathbf{k})+X_L(\mathbf{k})\Psi_X^{(j)}(\mathbf{k}),\qquad(8)$$

$$\Psi_{UP}^{(j)}(\mathbf{k})=C_U(\mathbf{k})\Psi_{ph}^{(j)}(\mathbf{k})+X_U(\mathbf{k})\Psi_X^{(j)}(\mathbf{k}).\qquad(9)$$

The coefficients of the transformation (Hopfield coefficients) are chosen in order to diagonalize the quadratic part of the Hamiltonian.

The interaction term in the polariton basis looks quite cumbersome: alongside with terms describing the scattering of polaritons within the lower polariton branch (LPB) and the upper polariton branch (UPB), it contains interbranch scattering terms. However, it can be simplified if the splitting between polariton branches (Rabi splitting) is much larger than the detuning between the energy of the lower polaritons and the excitation energy. In this case, only the LPB is excited and the UPB can be neglected.[53] In the $\mathbf{k}$ representation for lower polariton field, one obtains the following system of two coupled nonlinear integral equations:

$$\left[i\frac{\partial}{\partial t}-E_{LP}(\mathbf{k})\right]\Psi_{LP}^{j}(\mathbf{k})=\int\alpha_1(\mathbf{k},\mathbf{k}',\mathbf{q})\Psi_{LP}^{(j)*}(\mathbf{k}')\Psi_{LP}^{(j)}(\mathbf{k}-\mathbf{q})\times\Psi_{LP}^{(j)}(\mathbf{k}'+\mathbf{q})d\mathbf{k}'d\mathbf{q}+\int\alpha_2(\mathbf{k},\mathbf{k}',\mathbf{q})\Psi_{LP}^{(\bar{j})*}(\mathbf{k}')\times\Psi_{LP}^{(\bar{j})}(\mathbf{k}-\mathbf{q})\Psi_{LP}^{(j)}(\mathbf{k}'+\mathbf{q})d\mathbf{k}'d\mathbf{q}+\delta(\mathbf{k})F_je^{-i\omega_0 t},\qquad(10)$$

where

$$E_{LP}(\mathbf{k})=\frac{E_X(\mathbf{k})+E_{ph}(\mathbf{k})-\sqrt{[E_X(\hat{\mathbf{k}})+E_{ph}(\mathbf{k})]^2+V_R^2}}{2}\qquad(11)$$

is the bare LPB. The effective matrix element of polariton-polariton scattering reads

$$\alpha_{1,2}(\mathbf{k},\mathbf{k}',\mathbf{q})=W_{1,2}X_L^*(\mathbf{k})X_L^*(\mathbf{k}')X_L(\mathbf{k}-\mathbf{q})X_L(\mathbf{k}'+\mathbf{q})\qquad(12)$$

and the pumping amplitude for LPB polaritons is

$$F_j=C_L(0)P_j.\qquad(13)$$

Going back to the direct space, one obtains the following set of integral equations:

$$\left[i\frac{\partial}{\partial t}-E_{LP}(\hat{\mathbf{k}})\right]\Psi_{LP}^{(j)}(\mathbf{x})=\int U_1(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')\Psi_{LP}^{(j)*}(\mathbf{x}')\times\Psi_{LP}^{(j)}(\mathbf{x}'')\Psi_{LP}^{(j)}(\mathbf{x}''')d\mathbf{x}'d\mathbf{x}''d\mathbf{x}'''+\int U_2(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')\Psi_{LP}^{(\bar{j})*}(\mathbf{x}')\times\Psi_{LP}^{(\bar{j})}(\mathbf{x}'')\Psi_{LP}^{(j)}(\mathbf{x}''')d\mathbf{x}'d\mathbf{x}''d\mathbf{x}'''+F_je^{-i\omega_0 t},\qquad(14)$$

where the kernels $U_1(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')$ and $U_2(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')$ describe lower polariton scattering in the direct space,

$$U_{1,2}(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')=\frac{1}{(2\pi)^9}\int\alpha_{1,2}(\mathbf{k},\mathbf{k}',\mathbf{q})\times e^{i[\mathbf{k}(\mathbf{x}-\mathbf{x}'')+\mathbf{k}'(\mathbf{x}'-\mathbf{x}''')+\mathbf{q}(\mathbf{x}''-\mathbf{x}''')]}d\mathbf{k}d\mathbf{k}'d\mathbf{q}.\qquad(15)$$

Note that due to the complex $\mathbf{k},\mathbf{k}'$ dependence of the Hopfield coefficients X_L, polariton-polariton interactions are, in general, nonlocal in the direct space, $U_{1,2}(\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''')\neq U_{1,2}(\mathbf{x},\mathbf{x}')\delta(\mathbf{x}-\mathbf{x}'')\delta(\mathbf{x}'-\mathbf{x}''')$. The locality holds only if polaritons are created in a narrow region around $k=0$ where

Hopfield coefficients can be approximated by a constant. In this case, the dynamics of the polariton system can be described by a set of spinor Gross-Pitaevskii equations,[12]

$$\left[i\frac{\partial}{\partial t}-E_{LP}(\hat{\mathbf{k}})\right]\Psi^{(j)}(\mathbf{x},t)=[\alpha_1|\Psi^{(j)}(\mathbf{x},t)|^2 + \alpha_2|\Psi^{(\bar{j})}(\mathbf{x},t)|^2]\Psi^{(j)}(\mathbf{x},t) + F_j e^{-i\omega_0 t}. \quad (16)$$

The ratio α_1/α_2 characterizes the anisotropy of the polariton-polariton interactions. In the isotropic case, $\alpha_1/\alpha_2=1$. According to theoretical estimations by Ciuti *et al.*[54] for QW excitons, usually $|\alpha_1|\gg|\alpha_2|$. From fitting the experimental data on polarized emission of light from microcavities, this ratio was estimated as $\alpha_1/\alpha_2\approx-20$.[56,55]

III. DISPERSION OF ELEMENTARY EXCITATIONS

Now, let us find the dispersion of elementary excitations of an interacting polariton system under a monomode pumping at $k=0$. In the case of BEC of polaritons with infinite lifetime at thermal equilibrium, polariton condensate is linearly polarized and its spectrum consists of two Bogoliubov-like branches with slightly different sound velocities.[12] However, in the case of quasiresonant cw pumping and for finite polariton lifetime, the situation can be qualitatively different.

First, the polarization of the macroscopically occupied polariton mode is not chosen by the system itself but is determined by the pump. The relation is not, however, straightforward, as shown in Ref. 43: the anisotropy of polariton-polariton interactions leads to the deviation of the polarization of this mode from the polarization of the pump.

Second, the dispersion of elementary excitations differs strongly from the Bogoliubov-like one. In the scalar model, the dispersion was shown to contain a flat region near $k=0$,[17,18,48,50] and one can expect a similar result if spin is taken into account.

To find the response of the polariton system, we use the procedure similar to those used in the Refs. 11, 12, 18, and 43. Under the monomode pump at $k=0$, the solution of the spinor Gross-Pitaevskii equation can be recast in the following form:

$$\vec{\Psi}=e^{-i\omega_0 t}[\overrightarrow{\Psi_0}+\vec{A}e^{i(\mathbf{kr}-\omega t)}+\vec{B}^{*}e^{-i(\mathbf{kr}-\omega^{*}t)}], \quad (17)$$

where $\vec{\Psi}=(\Psi^{\uparrow};\Psi^{\downarrow})$, $\overrightarrow{\Psi_0}$ is the amplitude of the pumped state (the macro-occupied mode), and $\vec{A}=(A^{\uparrow};A^{\downarrow})$ and $\vec{B}=(B^{\uparrow};B^{\downarrow})$ are the amplitudes of the excitations with wave vector **k** and frequency ω. Note that the frequency of the macro-occupied mode ω_0 in our case is fixed by the pump and is not determined by the concentration-dependent blueshift as in the case of polariton BEC at thermal equilibrium.[12] This is the reason why the dispersions differ qualitatively in these two cases.

To find the dispersions of elementary excitations, one should insert Eq. (17) into Eq. (16) and perform the linearization with respect to the excitation amplitudes $\vec{A}$ and $\vec{B}$. The latter means that $|\vec{A}|,|\vec{B}|\ll|\overrightarrow{\Psi_0}|$, assuming that interactions are weak enough and the macro-occupied mode remains weakly depleted. A straightforward algebra gives the following set of six coupled algebraic equations for $\overrightarrow{\Psi}_0,\vec{A},\vec{B}$:

$$\left[\left(E_{LP}(0)-\omega_0-\frac{i}{\tau}\right)+(\alpha_1|\Psi_0^{\uparrow}|^2+\alpha_2|\Psi_0^{\downarrow}|^2)\right]\Psi_0^{\uparrow}+F_{\uparrow}=0, \quad (18)$$

$$\left\{\left[E_{LP}(0)-\omega_0-\frac{i}{\tau}\right]+(\alpha_1|\Psi_0^{\downarrow}|^2+\alpha_2|\Psi_0^{\uparrow}|^2)\right\}\Psi_0^{\downarrow}+F_{\downarrow}=0, \quad (19)$$

$$\left[E_{LP}(0)-\omega_0-\omega-\frac{i}{\tau}+2\alpha_1|\Psi_0^{\uparrow}|^2+\alpha_2|\Psi_0^{\downarrow}|^2\right]A^{\uparrow}+\alpha_1\Psi_0^{\uparrow 2}B^{\uparrow} + \alpha_2\Psi_0^{\uparrow}\Psi_0^{\downarrow *}A^{\downarrow}+\alpha_2\Psi_0^{\uparrow}\Psi_0^{\downarrow}B^{\downarrow}=0, \quad (20)$$

$$\alpha_1\Psi_0^{\uparrow 2}A^{\uparrow}+\left[E_{LP}(0)-\omega_0+\omega+\frac{i}{\tau}+2\alpha_1|\Psi_0^{\uparrow}|^2+\alpha_2|\Psi_0^{\downarrow}|^2\right]B^{\uparrow} + \alpha_2\Psi_0^{\uparrow *}\Psi_0^{\downarrow *}A^{\downarrow}+\alpha_2\Psi_0^{\uparrow *}\Psi_0^{\downarrow}B^{\downarrow}=0, \quad (21)$$

$$\alpha_2\Psi_0^{\uparrow *}\Psi_0^{\downarrow}A^{\uparrow}+\alpha_2\Psi_0^{\uparrow}\Psi_0^{\downarrow}B^{\uparrow}+\left[E_{LP}(0)-\omega_0-\omega-\frac{i}{\tau} + 2\alpha_1|\Psi_0^{\downarrow}|^2+\alpha_2|\Psi_0^{\uparrow}|^2\right]A^{\downarrow}+\alpha_1\Psi_0^{\downarrow 2}B^{\downarrow}=0, \quad (22)$$

$$\alpha_2\Psi_0^{\uparrow *}\Psi_0^{\downarrow *}A^{\uparrow}+\alpha_2\Psi_0^{\uparrow *}\Psi_0^{\downarrow}B^{\uparrow}+\alpha_1\Psi_0^{\downarrow 2}A^{\downarrow}+\left[E_{LP}(0)-\omega_0+\omega + \frac{i}{\tau}+2\alpha_1|\Psi_0^{\downarrow}|^2+\alpha_2|\Psi_0^{\uparrow}|^2\right]B^{\downarrow}=0, \quad (23)$$

where τ is the lifetime of the cavity polaritons. The first two of these equations determine the state of the macro-occupied mode, while the other four yield the dispersions of its elementary excitations. It is easily seen from Eqs. (18) and (19) that polarization of the pumped mode differs from the polarization of the pump, in general. Indeed, one has

$$\frac{\Psi_0^{\uparrow}}{\Psi_0^{\downarrow}}=-\frac{F^{\uparrow}}{F^{\downarrow}}\frac{\left(E_{LP}(0)-\omega_0-\frac{i}{\tau}\right)+(\alpha_1|\Psi_0^{\uparrow}|^2+\alpha_2|\Psi_0^{\downarrow}|^2)}{\left(E_{LP}(0)-\omega_0-\frac{i}{\tau}\right)+(\alpha_1|\Psi_0^{\downarrow}|^2+\alpha_2|\Psi_0^{\uparrow}|^2)}. \quad (24)$$

It follows from Eq. (24) that the polarizations of the pump and the polariton system coincide only in the case of the circularly polarized pump. On the other hand, for an elliptical pumping, the polarizations of the polariton system and the pump can strongly differ. First, due to the effect of the self-induced Larmor precession,[38] the polarization ellipse of the condensate is rotated as compared to the polarization ellipse of the pump by an angle which depends on the pumping intensity. Second, the circular polarization degree of the condensate differs from that of the pump due to the different blueshifts for right and left circularly polarized components.

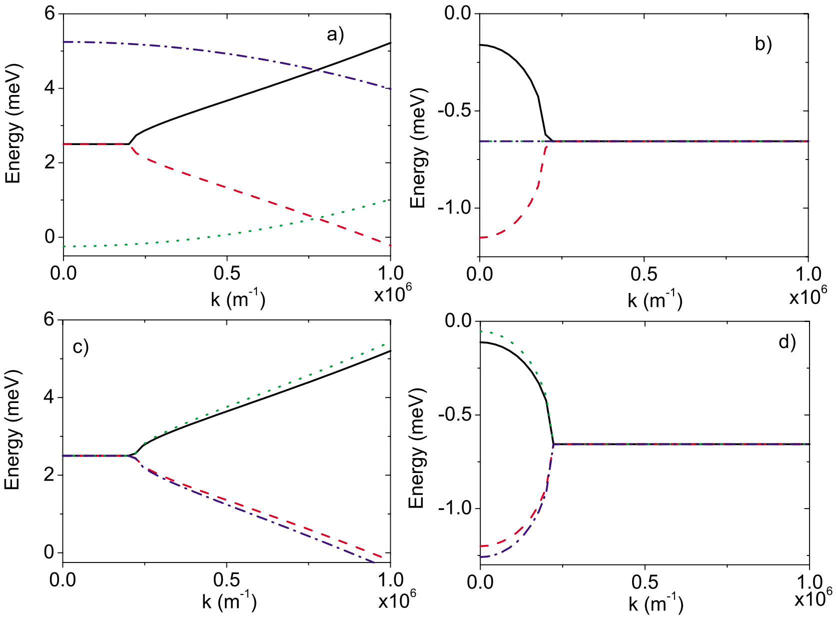

FIG. 1. (Color online) [(a) and (c)] Real and [(b) and (d)] imaginary parts of dispersions of the excitations in the case of [(a) and (b)] circular and [(c) and (d)] linear polarizations of the macroscopically occupied mode with a flat part at $k=0$.

The interplay between the polarizations of the pump and pumped mode was analyzed in detail in Ref. 43.

Now, let us turn to the analysis of the dispersions of the elementary excitations of the driven mode. Although corresponding analytical expressions can be obtained in the general case of the elliptical polarization, they are quite cumbersome. These expressions simplify a lot, however, in the case of the circular and linear polarization of the driven mode.

In the case of a circularly polarized σ^+ pumping, the elementary excitations are also circularly polarized. The four dispersion branches are described by the following formulas:

$$\omega^{\uparrow}_{1,2} = \omega_0 - \frac{i}{\tau} \pm \sqrt{[E_{LP}(k) - \omega_0 + 2\alpha_1 n]^2 - (\alpha_1 n)^2}, \quad (25)$$

$$\omega^{\downarrow}_{1} = -\frac{i}{\tau} + E_{LP}(k) + \alpha_2 n, \quad (26)$$

$$\omega^{\downarrow}_{2} = 2\omega_0 - \frac{i}{\tau} - E_{LP}(k) - \alpha_2 n, \quad (27)$$

where $n=|\overrightarrow{\Psi_0}|^2$

The dependencies of real and imaginary parts of $\omega^{\uparrow,\downarrow}_{1,2}(k)$ are shown in Figs. 1(a) and 1(b) and 2(a) and 2(b), respectively. Figure 1 shows the flat dispersion at $k=0$, whereas Fig. 2 shows the flat dispersion at $k\neq 0$. The bare polariton dispersion $E_0(k)$ is taken parabolic with a polariton mass given by $m_{pol}=3\times 10^{-5} m_0$, where m_0 is the free electron mass. We use $\alpha_1=6xE_b a_B^2/S$, where $a_B=100$ Å is the two dimensional exciton Bohr radius, $E_b=8$ meV is the exciton binding energy, $x=1/4$ is the squared exciton fraction, and $S=100\ \mu\text{m}^2$ is the laser spot area. The polariton lifetime is $\tau=2$ ps. These parameters are typical for a GaAlAs microcavity. We take $n=1.1\times 10^4, 2.5\times 10^3$. It is seen from Eqs.

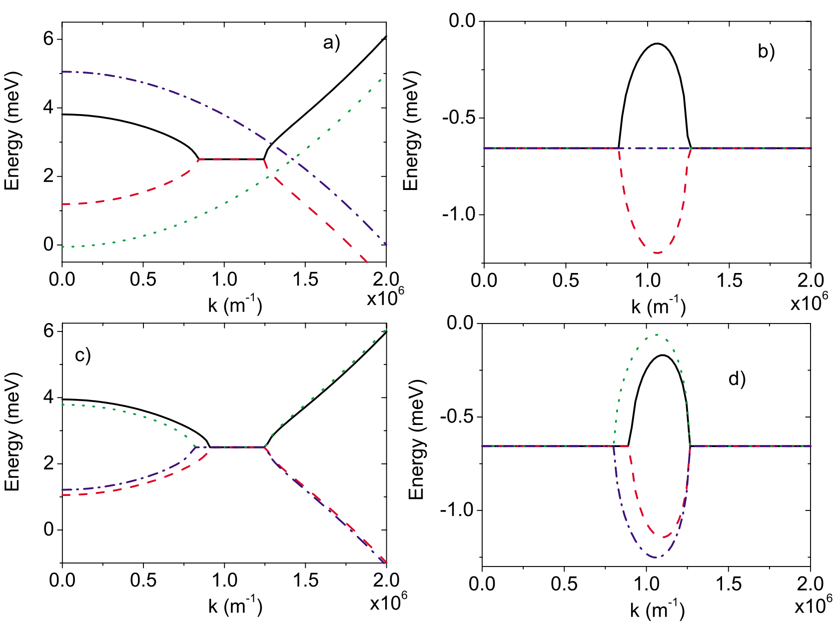

FIG. 2. (Color online) [(a) and (c)] Real and [(b) and (d)] imaginary parts of dispersions of the excitations in the case of [(a) and (b)] circular and [(c) and (d)] linear polarization of the macroscopically occupied mode with a flat part at $k\neq 0$.

(26) and (27) that renormalization of the dispersion of cross-polarized excitations consists only in the concentration-dependent shift with respect to the bare dispersion. It remains parabolic with a constant imaginary part given by $1/\tau$. This is because polariton-polariton interactions do not mix the circularly polarized components.

The renormalization of the copolarized dispersion is much more interesting. It follows from Eq. (25) that its real part is dipsersionless in the vicinity of the point where $E_{LP}=\omega_0+2\alpha_1 n$. Physically, it means that the renormalized mode is fully diffusive.

In the case of a linearly polarized driven mode, the elementary excitations are also linearly polarized with dispersions given by the following expressions:

$$\omega_{1,2}^{co}=\omega_0-\frac{i}{\tau}\pm\sqrt{[E_{LP}(k)-\omega_0+\alpha_1 n+\alpha_2 n]^2-\frac{1}{4}(\alpha_1 n+\alpha_2 n)^2}, \quad (28)$$

$$\omega_{1,2}^{cross}=\omega_0-\frac{i}{\tau}\pm\sqrt{[E_{LP}(k)-\omega_0+\alpha_1 n]^2-\frac{1}{4}(\alpha_1 n-\alpha_2 n)^2}. \quad (29)$$

As linear polarizations are mixed by anisotropic polariton-polariton interactions, the dispersions of both co- and cross-polarized modes contain flat parts (purely dissipative regions). Real and imaginary parts of the dispersions in the case of linearly polarized mode are shown in Figs. 1(c) and 1(d) (flat at $k=0$) and 2(c) and 2(d) (flat at $k\neq 0$), respectively. Here, we took $n=2.5\times 10^4, 5\times 10^3$. The difference in behavior of co- and cross-polarized components is governed by the value and the sign of α_2. We have considered a realistic case with $\alpha_2=-0.1\alpha_1$.

IV. STABILITY ANALYSIS

The approach presented above is only valid for a single macroscopically occupied polariton quantum state. This is indeed the case if the imaginary parts of the eigenfrequencies of all excited states are negative. On the other hand, if the imaginary part is positive for any of the states, the scattering toward this state becomes stimulated and the state itself becomes macroscopically occupied. In this section, we analyze the stability of the polariton dispersions obtained in the previous sections. As before, we shall concentrate on two cases, namely, the circularly polarized mode and the linearly polarized mode. In the further analysis, the most important parameter will be the detuning $\Delta_k=\omega_0-E_{LP}(k)$.

A. Circularly polarized mode

For a strictly circular pump, only the coefficient α_1 is important because the amplitude of the cross-polarized component of the polariton state is strictly zero. The driven mode is circularly polarized. The stability condition in this case reads

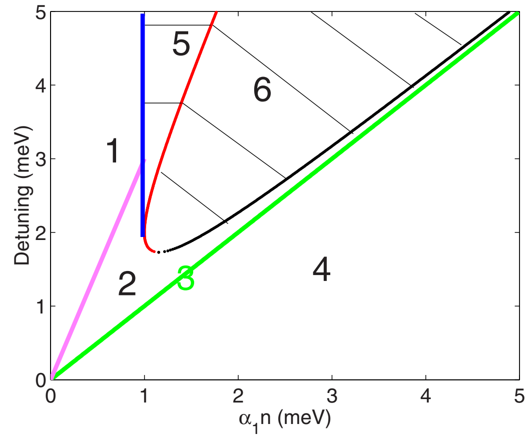

FIG. 3. (Color online) Regions of stability for the circularly polarized mode: (1) flat part of the spectrum for $(+k,-k)$ states, (2) flat part of the spectrum for $k=0$, (3) linear spectrum in $k=0$, (4) parabolic spectrum, (5) instability of the $(+k,-k)$ states, and (6) instability of the ground state.

$$3(\alpha_1 n)^2-4\Delta_k\alpha_1 n+\Delta_k^2+\frac{1}{\tau^2}>0. \quad (30)$$

Therefore, for a given value of Δ_k, the system is unstable against parametric scattering in the $(+k,-k)$ states if

$$\alpha_1 n\in\left[\frac{2}{\sqrt{3}\tau},\frac{2\Delta_0-\sqrt{\Delta_0^2-3/\tau^2}}{3}\right]. \quad (31)$$

The ground state itself is unstable if

$$\alpha_1 n\in\left[\frac{2\Delta_0-\sqrt{\Delta_0^2-3/\tau^2}}{3},\frac{2\Delta_0+\sqrt{\Delta_0^2-3/\tau^2}}{3}\right]. \quad (32)$$

The dispersion shows flat parts if the stability condition is verified and if

$$3(\alpha_1 n)^2-4\Delta_k\alpha_1 n+\Delta_k^2<0, \quad (33)$$

which occurs when $\alpha_1 n\in[\frac{\Delta_k}{3},\Delta_k]$. Therefore, if $\alpha_1 n$ is between 0 and $\Delta_0/3$, the flat parts are present at nonzero wave vectors, and if $\alpha_1 n$ is between $\Delta_0/3$ and Δ_0, this flat part is around $k=0$. The condition $\alpha_1 n=\Delta_0$ yields a linear spectrum. One should note that this line belongs to the stable region. All these results are summarized in Fig. 3. We took $1/\tau=1$ meV. The figure shows six different regions, two of which are unstable and four correspond to different types of dispersions. In region 1, the dispersion shows a flat part at some nonzero wave vector k (and $-k$); in region 2, the flat part is centered at $k=0$; line 3 has a Bogoliubov-like dispersion (linear at small wave vectors); region 4 has the original parabolic dispersion; regions 5 and 6 are unstable against parametric scattering and the dispersion should be analyzed by another method.

B. Linearly polarized mode

Here, we consider a linearly polarized macro-occupied mode. This situation can correspond to different pumping polarizations, including, but not limited to, the linear polarization. The stability of the system requires the following conditions to be fulfilled:

$$\frac{1}{\tau^2}+\Delta_k^2-2\Delta_k\alpha_1 n+n^2\left[\alpha_1^2-\frac{(\alpha_1-\alpha_2)^2}{4}\right]>0, \quad (34)$$

$$\frac{1}{\tau^2}+\Delta_k^2-2(\alpha_1+\alpha_2)\Delta_k n+n^2\left[\alpha_1^2+\alpha_2^2-\frac{(\alpha_1+\alpha_2)^2}{4}\right]>0. \quad (35)$$

These two conditions are equivalent if $\alpha_2=0$. If α_2 is positive, the condition of stability of the component copolarized with the macro-occupied mode is the strictest, and therefore it is sufficient to check only this condition. If α_2 is negative, which is the realistic case, the condition on the cross-polarized component becomes stronger. This conclusion agrees well with the current understanding of the spin-dependent polariton-polariton scattering. Indeed, it agrees with the fact that two polaritons of a given linear polarization will scatter preferentially toward cross-polarized polariton states if α_2 is negative and copolarized if α_2 is positive.[38] Therefore, in the case of negative α_2, elementary excitations of the macro-occupied mode are mainly cross polarized and their stability governs the stability of the whole system. In what follows, we study only the realistic case and check the stability of the cross-polarized component. The system is always stable if

$$\Delta_k<\Delta_{min}=\frac{1}{\tau}\sqrt{4\frac{\alpha_1^2}{(\alpha_1-\alpha_2)^2}-1}. \quad (36)$$

Above this value, the stability region is limited by the values

$$n_\pm=\frac{\alpha_1\Delta_k\pm 1/2(\alpha_1-\alpha_2)\sqrt{\Delta_k^2-\Delta_{min}^2}}{\alpha_1^2-1/4(\alpha_1-\alpha_2)^2}. \quad (37)$$

States with finite wave vectors show flat dispersions on the range

$$n\in\left[0,\frac{\Delta_0(\alpha_1+\alpha_2)}{2\alpha_1^2-1/2(\alpha_1-\alpha_2)^2}\right] \quad (38)$$

and the $k=0$ state has flat dispersion on the range

$$\left[\frac{\Delta_0(\alpha_1+\alpha_2)}{2\alpha_1^2-1/2(\alpha_1-\alpha_2)^2},\frac{\Delta_0(3\alpha_1-\alpha_2)}{2\alpha_1^2-1/2(\alpha_1-\alpha_2)^2}\right]. \quad (39)$$

The flat dispersion for the copolarized component occurs in the range

$$\left[\Delta_0\frac{(\alpha_1+\alpha_2)-\sqrt{2\alpha_1\alpha_2+1/4(\alpha_1+\alpha_2)^2}}{\alpha_1^2+\alpha_2^2-1/4(\alpha_1+\alpha_2)^2};\right.$$
$$\left.\Delta_0\frac{(\alpha_1+\alpha_2)+\sqrt{2\alpha_1\alpha_2+1/4(\alpha_1+\alpha_2)^2}}{\alpha_1^2+\alpha_2^2-1/4(\alpha_1+\alpha_2)^2}\right]. \quad (40)$$

The result is plotted in Fig. 4. This figure is more complicated than Fig. 3 because both co- and cross-polarized components can show different types of dispersions. In region 1, the dispersion shows a flat part at some nonzero wave vector $\pm k$ [this applies to both polarizations, as demonstrated by Fig. 1(c)]; in regions 2 and 2′, the flat part is centered at

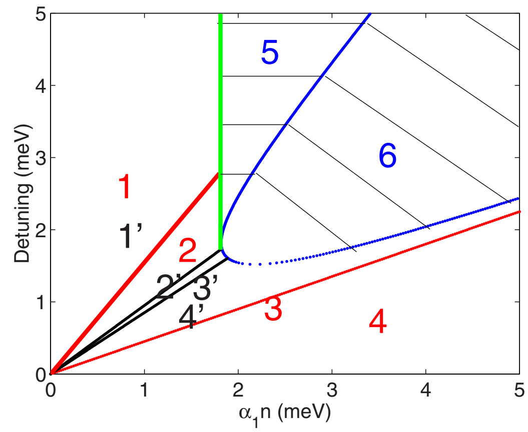

FIG. 4. (Color online) Regions of stability for the linearly polarized mode: (1) flat part of the spectrum for $(+k,-k)$ states, $(2,2')$ flat part of the spectrum for $k=0$, $(3,3')$ linear spectrum in $k=0$, $(4,4')$ parabolic spectrum, (5) instability of the $(+k,-k)$ states, and (6) instability of the ground state.

$k=0$ (the prime corresponds to the cross-polarized component); lines 3 and 3′ have the Bogoliubov-like dispersion, which takes place at different conditions for the co- and cross-polarized components; region 4 has an ordinary parabolic dispersion; regions 5 and 6 are unstable. It is important to underline that the results shown here have been obtained for the linearly polarized macro-occupied polariton mode[43] and not necessarily for a linearly polarized pumping.

V. EFFECT OF THE RENORMALIZATION OF POLARITON DISPERSION ON THE EMISSION SPECTRA OF MICROCAVITIES

The renormalization of the dispersion of elementary excitations of microcavity under resonant pumping obtained and analyzed analytically in the previous sections can be illustrated by the emission spectra in the pump-probe geometry. We perform numerical simulation of such an experiment using the coupled Gross-Pitaevskii equation for excitons and Schrödinger equation for photons taking into account their polarization[12] which are solved in time domain. The polariton lifetime is taken equal to 1 ps. The circularly polarized pump is spatially homogeneous and detuned by 2.5 meV from the bottom of the low polariton branch. A weak probe of 0.1 ps duration and 1 μm spatial size is sent 15 ps after the pump is turned on. This probe should excite a large part of the excitation dispersion. The photon component of the wave function obtained as a solution of the above mentioned equations is Fourier transformed over 100 ps to find the dispersions. Figure 5 shows resulting dispersions obtained by increasing the pump intensity. Figure 5(a) shows dispersion with flat regions at ±2 μm^{-1} (region 1 of Fig. 3). One can see the bright emission spots due to the renormalization of the imaginary part of the dispersion. Figure 5(b) shows flat dispersion centered at $k=0$ (region 2 of Fig. 3). Figure 5(c) shows the Bogoliubov-like dispersion linear at small wave vectors (line 3 of Fig. 3). Finally, Fig. 5(d) shows the parabolic dispersion corresponding to region 4 of Fig. 3. This

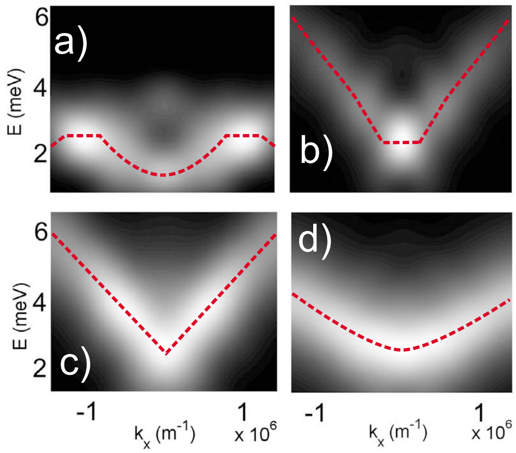

FIG. 5. (Color online) Emission spectra showing renormalized dispersions in the order of increasing pumping power: (a) flat at $k \neq 0$, (b) flat at $k=0$, (c) linear, and (d) parabolic. The red lines show the real parts of the theoretical dispersions calculated using Eq. (25).

gives a full set of different types of dispersions of excitations that can be obtained in a quasiresonantly driven system of polaritons, described above by analytical means.

The linear spectrum of the mode copolarized with an excitation means that one can expect the appearance of superfluidity in the resonantly driven system. However, this result holds only if the processes of the relaxation of circular polarization (e.g., TE-TM splitting) are nonrelevant. Indeed, the latter will lead to the transitions between two circularly polarized branches. As the cross-polarized branch always remains parabolic and lies below the copolarized branch, the interbranch transitions induced by spin relaxation will lead to the suppression of superfluidity according to the Landau criterion. This corrects the previous result of Carusotto and Ciuti[11] obtained for the case of spinless cavity polaritons.

The direct proof for superfluidity of a liquid can be obtained only in propagation experiments which would show zero viscosity (no dissipation). In order to reveal the superfluid regime for the ensemble of exciton polaritons, we have modeled the propagation of a Gaussian-shape repulsive potential fluctuation (defect) of 7 μm diameter and 10 meV amplitude propagating at a speed of 5 μm/ps through the polariton condensate in two regimes: (1) when the pumping is such as to make the polariton dispersion parabolic at $k=0$ [Fig. 5(d)] and (2) when the pumping is chosen to provide a linear dispersion of excitations in the vicinity of $k=0$ [Fig. 5(c)]. The results are shown in Fig. 6. If the dispersion is parabolic [Fig. 6(a)], the propagation of the defect induces the supplementary excitations leading to the polariton density waves. They are visible about 50 μm away from the defect, even though the lifetime in the system is quite short (1 ps). However, if the pumping intensity is chosen to yield the linear dispersion [Fig. 6(b)], no density waves are seen even in the vicinity of the defect. This is a clear indication that the polaritons are not perturbed by the motion of a defect, and therefore the moving body does not lose its energy by interaction with the polaritons. This is characteristic for the dissipationless propagation of a body through a

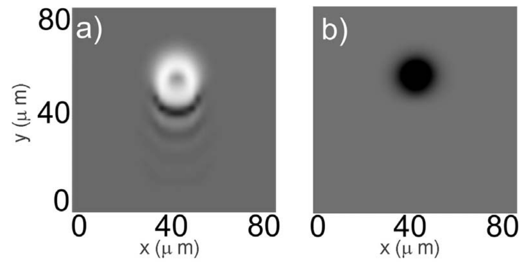

FIG. 6. Motion of a defect through (a) normal polaritons and (b) superfluid polaritons. Logarithmic scale intensity grayscale map.

superfluid. We have checked also that the total density of polaritons does not depend on the fact whether the defect is moving or not, in the case the linear dispersion. As the energy lost by the system per a unit of time is directly proportional to the density of polaritons in the system, this confirms that there is no additional dissipation linked with the motion of the object.

From the experimental point of view, it would be interesting to work with the linearly polarized probe orthogonal to the pump. In this configuration, it should be easier to detect the system response to a weak probe.

VI. CONCLUSIONS

In conclusion, we have analyzed the response of a microcavity to a coherent cw pump taking into account the polarization degree of freedom. The dispersion of a pumped cavity mode is, in general, different from the linear Bogoliubov-like dispersion characteristic for systems of weakly interacting bosons. This is a consequence of the fact that our resonantly driven system is out of thermal equilibrium. The dispersion can show flat diffusive regions at nonzero wave vectors or at $k=0$. The linear Bogoliubov-like dispersion can be recovered at some particular conditions, namely, when the blueshift exactly compensates the detuning. This result is important since it proves that the finite lifetime of the particles involved is not, in principle, an obstacle to the generation of a superfluid. We have performed an analysis of the stability of the macro-occupied mode in the cases of circular and linear polarizations. The numerical simulations in the pump-probe geometry demonstrate four different types of dispersions for the excitations of the macro-occupied mode. In the case of a linear dispersion, the polaritons form a radiatively decaying superfluid, showing a vanishing mechanical viscosity.

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